Violating Nyquist: Another Source of Significant Error in the Instrumental Temperature Record

William Ward, January 12, 2019

The instrumental temperature record provides historical surface air temperatures since the invention of the thermometer in 1714. However, 1850 marks the start of the generally accepted quasi-global record. As of 2018 we have 169 years of data in the global record. According to Wikipedia, the global average temperature has increased by 0.85 ± 0.20 °C in the period from 1880 to 2012. Therefore, the warming trend is 0.064 ± 0.015 °C per decade over that period.

The instrumental temperature record is built upon 2 measurements taken daily at each monitoring station, specifically the maximum temperature (Tmax) and the minimum temperature (Tmin). These daily readings are then averaged to calculate a daily mean temperature: Tmean = (Tmax+Tmin)/2. Tmax and Tmin measurements are also used to calculate monthly and yearly mean temperatures. These mean temperatures are then used to determine warming or cooling trends. The method of using Tmax and Tmin as the basis for further calculations will be referred to as the "historical method".

To one extent or another, the instrumental temperature record underpins most published climate research and the corresponding alarm over climate change that follows. Therefore, the validity of this research is dependent upon the accuracy of the record. If the record is not accurate then the research and corresponding climate alarmism must be called into question. So, are 2 daily measurements of temperature sufficient to accurately determine mean temperatures and temperature trends? This paper examines that question. It will be shown that the historical method is not sufficient to accurately calculate daily mean temperatures or long-term temperature trends. First, a proof will be presented, based upon the mathematics of signal processing. Then, actual temperature data from NOAA's USCRN (US Climate REFERENCE Network) will be used to demonstrate this empirically.

Signals and Sampling

This analysis of the instrumental temperature record utilizes the engineering discipline of signal analysis or signal processing. A **signal** is defined as an observable change in a measurable parameter that varies with respect to time. Here are some examples of signals:

1. Electrocardiogram (EKG): Shows the electrical activity of the heart, specifically a change in voltage with respect to time. See Figure 1.
2. Audio waveform: The change in air pressure as measured by a microphone, output as an electrical voltage with respect to time. See Figure 2.
3. S&P 500 stock market index: Percentage gain or loss vs. calendar date. See Figure 3.
4. Surface air temperature: Changes to temperature over time. See Figures 4A and 4B.

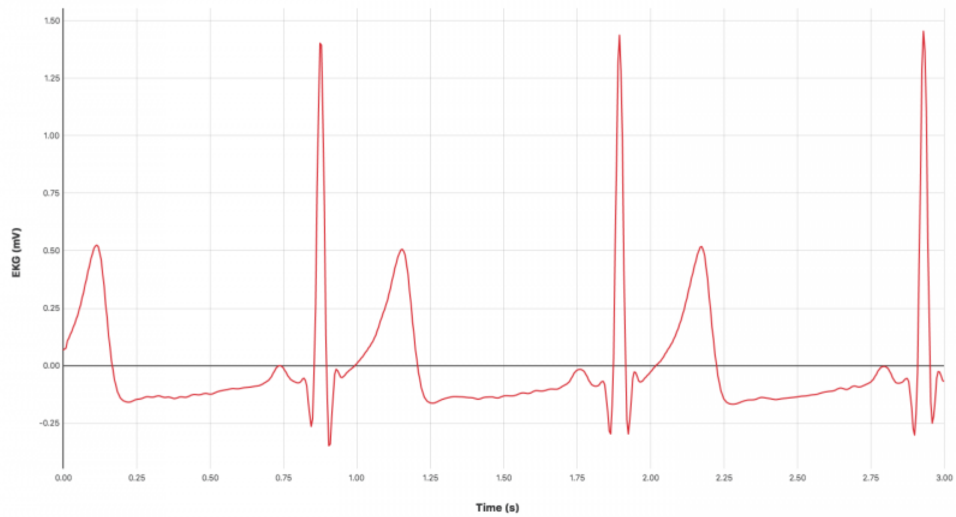

Figure 1: EKG Signal

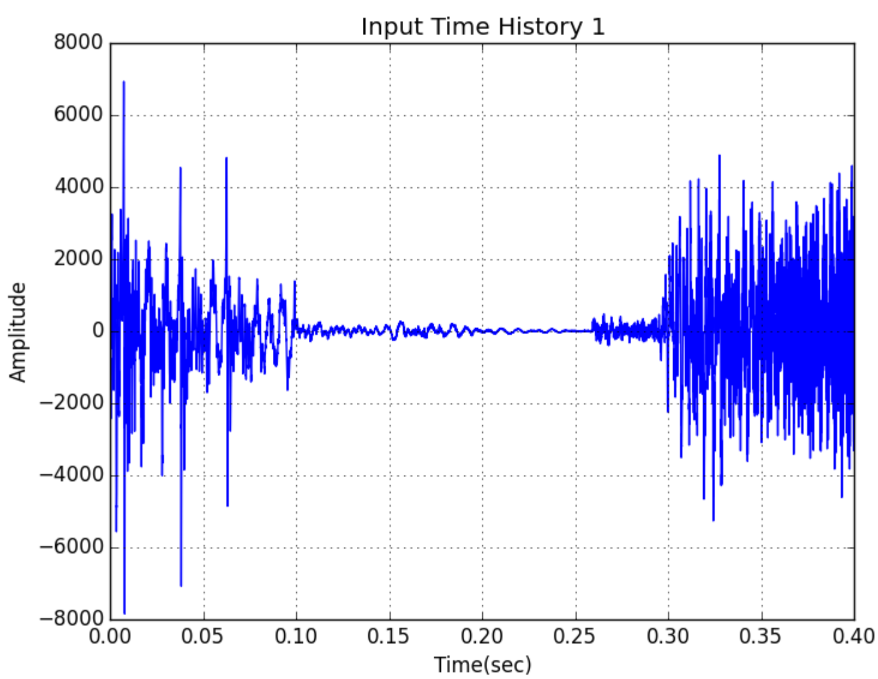

Figure 2: Audio Signal

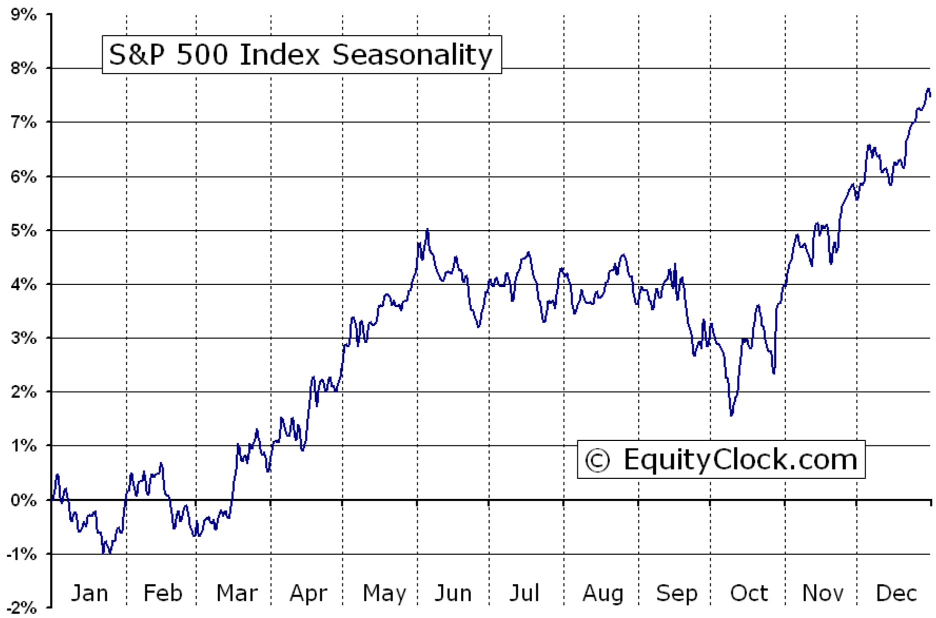

Figure 3: S&P 500 Index Signal

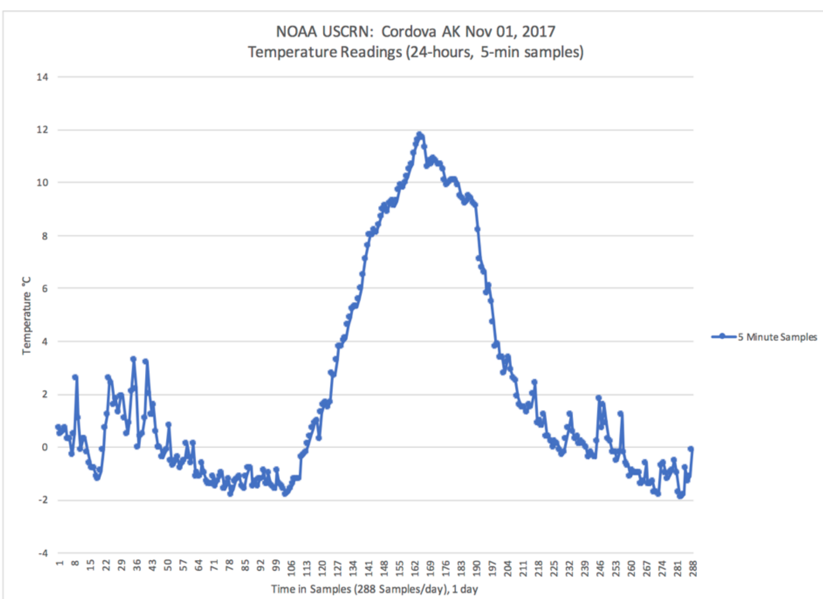

Figure 4A: Surface Air Temperature Signal, Cordova AK as measured on Nov 01, 2017

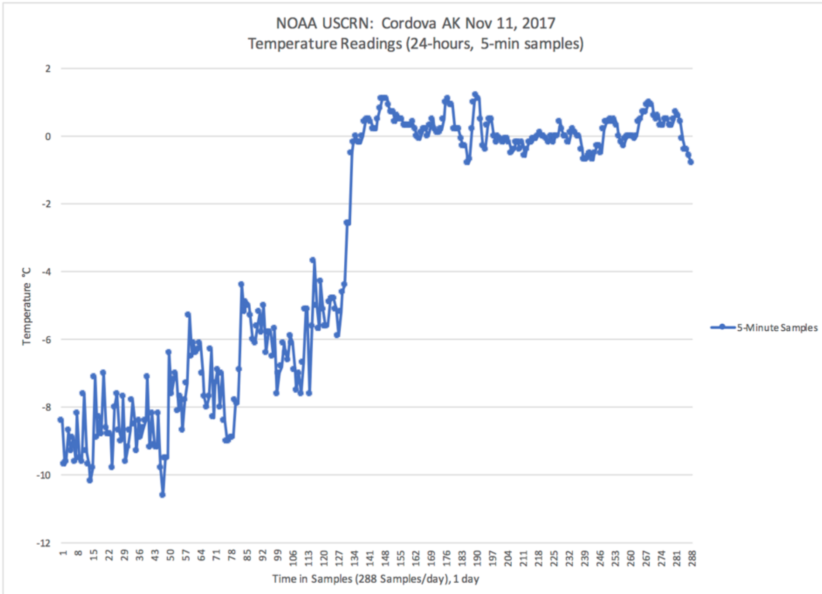

Figure 4B: Surface Air Temperature Signal, Cordova AK as measured on Nov 11, 2017

It is important to understand that **air temperature is a signal**, and if we want to measure it, we must abide by **the mathematical laws of signal processing**. The process of measuring a fixed parameter, such as the mass of a steel beam, is different from the process of measuring a signal. A single accurate and precise measurement of the beam's mass is sufficient, but a signal must be measured at a periodic rate with a process known as **sampling**. A signal that has a faster rate of change must be sampled more frequently than a signal with a slower rate of change. The question of exactly how frequently a signal must be sampled was answered in 1928 by Harry Nyquist, a Swedish-born American Electronics Engineer. A few years later, with the help of Claude Shannon, the Nyquist-Shannon Sampling Theorem was published. Nyquist tells us that our first requirement is to understand the "frequency content" or "bandwidth" of the signal we are sampling. We need to know what frequencies are contained in the signal; specifically, we need to know the highest frequency component of the signal. **According to the Nyquist-Shannon Sampling Theorem, we must sample the signal at a rate that is at least 2 times the highest frequency component of the signal.**

$f_s > 2B$

Where f_s is the sample rate or **Nyquist frequency** and B is the bandwidth or highest frequency component of the signal being sampled. **If we sample at the Nyquist Rate, we have captured all of the information available in the signal and it can be fully reconstructed. If we sample below the Nyquist Rate, the consequence is that our samples will contain error. As our sample rate continues to decrease below the Nyquist Rate, the error in our measurement increases.**

The Nyquist Sampling Theorem is **essential science** to every field of technology in use today. Digital audio, digital video, industrial process control, medical instrumentation, flight control systems, digital communications, etc., all rely on the essential math and physics of Nyquist. However, climate science and the instrumental temperature record completely ignore Nyquist and as a result all work derived from the record suffers from significant error. This calls into question any conclusions based upon the record.

Air Temperature Signal Frequency Content (Spectral Components of the Signal)

To begin our study of the instrumental temperature record, let's examine the frequency content of a hypothetical temperature signal. Then we can apply the concepts from Nyquist to examine the extent to which the record is corrupted with sampling error. This example is meant to illustrate the key components of a real signal, but it is important to understand that the signal for any given location over a particular period of time is unique and distinctive.

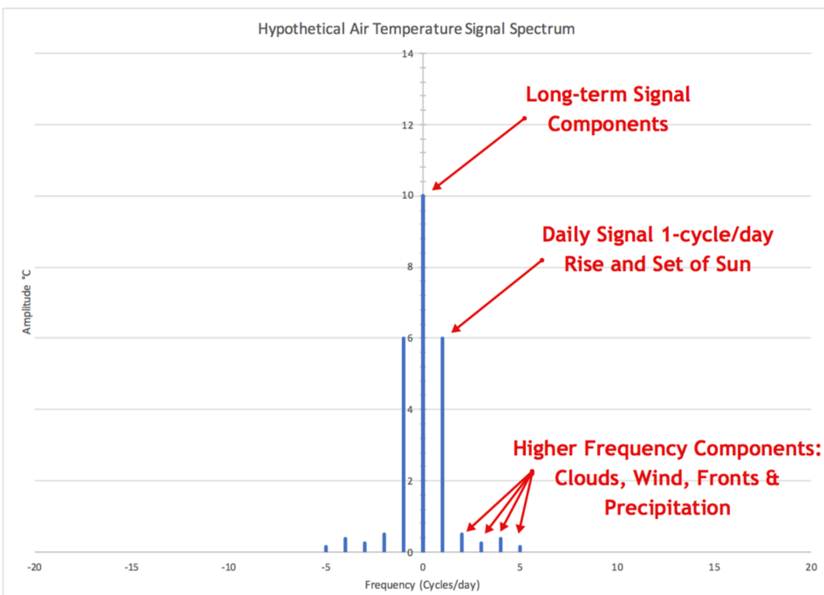

Figure 5A: Frequency Content (Spectrum) of Hypothetical Air Temperature Signal

Figure 5A plots the frequency components peak amplitude in °C against the frequency in cycles/day. Signals are almost always made up from a combination of other signals. This is true with air temperature, which has many components. The **"daily-signal"** is the intuitive signal component that is caused by the rising and setting of the sun every 24 hours, or once each day. This is shown at a frequency of 1-cycle/day. (Note: it is convention to show both "positive" and "negative" frequency in the graph, and the positive and negative sides are always mirror images of each other. The reason for this symmetry is beyond the scope of the discussion.) The peak amplitude of the daily-signal shown in the example of Figure 5A is 6°C, which represents a day where the daily fluctuation from the daily-signal varies 12°C peak-to-peak over the 24-hour period. The actual total daily variation may be greater or less than 12°C, based upon the effects of the **higher frequency components**, which are located further to the right of the daily-signal, on the positive x-axis. These correspond to various effects such as cloud cover changes, wind changes, weather front changes, precipitation changes, etc. These effects essentially modulate the daily-signal; they add to or subtract from the daily signal to arrive at the combined air temperature signal we measure. (Figure 6A gives a visual aid to show how signals of different frequencies add or subtract in the time domain to give a combined signal.)

To keep this example simple, frequencies above 5-cycles/day are not shown. An actual temperature signal will contain frequencies into the hundreds or thousands of cycles/day, but the amplitude of these higher frequency components diminishes as frequency increases. At some point these higher frequencies become negligible and will not meaningfully affect our sampling. The frequency content of each day is different, and different locations of the world will have different tendencies towards frequency content. Refer back to Figures 4A and 4B. Notice the different shapes of these 2 signals. They show temperature from the same station on 2 different days, 10-days apart. A study of these signals would show that they have different frequency spectrums. These differences will show themselves on a frequency chart with different values for the amplitudes of the components.

Frequency components can exist that are not integer multiples of the daily cycle. Content may exist that is 7.92-cycles/day for example. The example only shows integer multiples for the purpose of simple illustration.

Still referring to Figure 5A, let's look at the component located at 0-cycles/day. This is the very-low frequency content. There is more going on in this area of the figure than is apparent, because the scale of the chart does not allow us to see the actual detail that exists. Figure 5B shows the same example temperature signal, but with a magnified view ("zoomed-in") around 0-cycles/day. Note the values of the x-axis. Now, we can see more spectral content. This content appeared as a single line around 0-cycles/day in Figure 5A, due to the "zoomed-out" view. The outermost line, located at 1-cycle/365 days, with an amplitude of 10°C, represents the 1-year cycle frequency component or **"annual-signal"**. This tells us that the temperature varies 20°C peak-to-peak over the course of the year. This cycle is also very intuitive. As the Earth revolves around the sun, the apparent angle of the sun caused by the Earth's axial tilt creates the 4 seasons of winter, spring, summer and autumn. This process gives us the 1-year cycle component of our air temperature signal. As we progress inward toward 0-cycles/day, we next encounter the 10-year cycle frequency component. For simplicity, the number of cycles is being limited to 1-yr and 10-yr, but in actuality, content can exist at any frequency. The 10-year component represents an even longer-term variation that modulates the yearly signal. We know, for example, that over time winters may become colder or warmer and this is due to the longer-term cycle components. Finally, at the center we have the **"long-term signal"**, representing the variation over hundreds of thousands of years or longer. This would be the signal that drives the climate into and out of the glacial and interglacial periods of our current ice age (Quaternary). Practically speaking, it also represents the constant offset that all other observable signals see for the temperature scale being used (°C, °F, K).

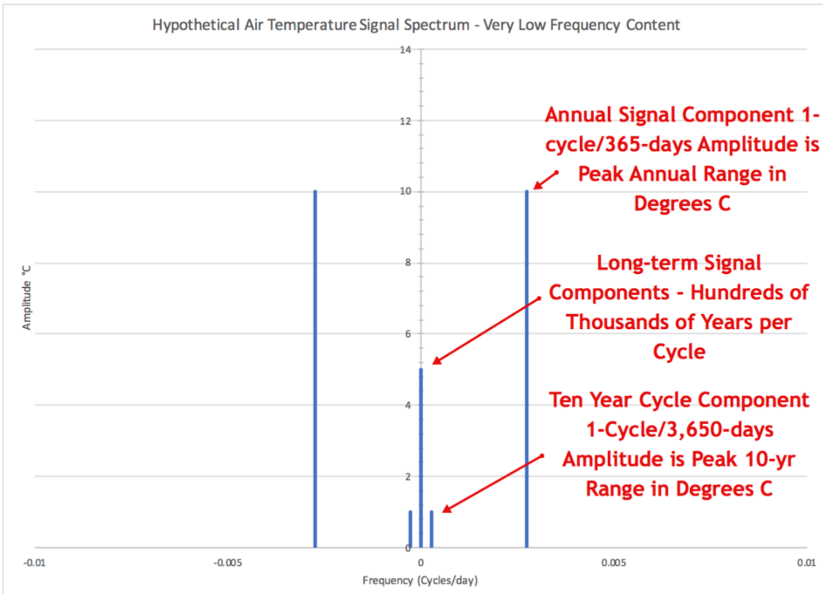

Figure 5B: Frequency Content (Spectrum) of Typical Air Temperature Signal – Low Frequency

Figure 6B shows an actual surface air temperature signal for Spokane, WA, for 365-days of 2008. The data is from NOAA's USCRN with temperature sampled every 5-minutes. The annual-signal is clearly visible, showing the low January temperatures, followed by rising temperatures in the spring, peak temperatures in the summer followed by falling temperatures into the autumn and next winter in December. The daily-signal, with subsequent higher-frequency components, can be seen "riding" on the annual-signal.

With these concepts at our disposal, now we are ready to examine the effects of using the historical method of sampling temperature 2 times a day and compare it to sampling at a rate specified by Nyquist. First, we will show mathematically and graphically in the frequency domain how sampling at 2-samples/day creates error. This may be a more abstract approach for some readers, but it is necessary to provide the proof. Then we will quickly get to a more intuitive approach that shows by example in the time domain just how sampling below the Nyquist Rate leaves us with an erroneous measurement.

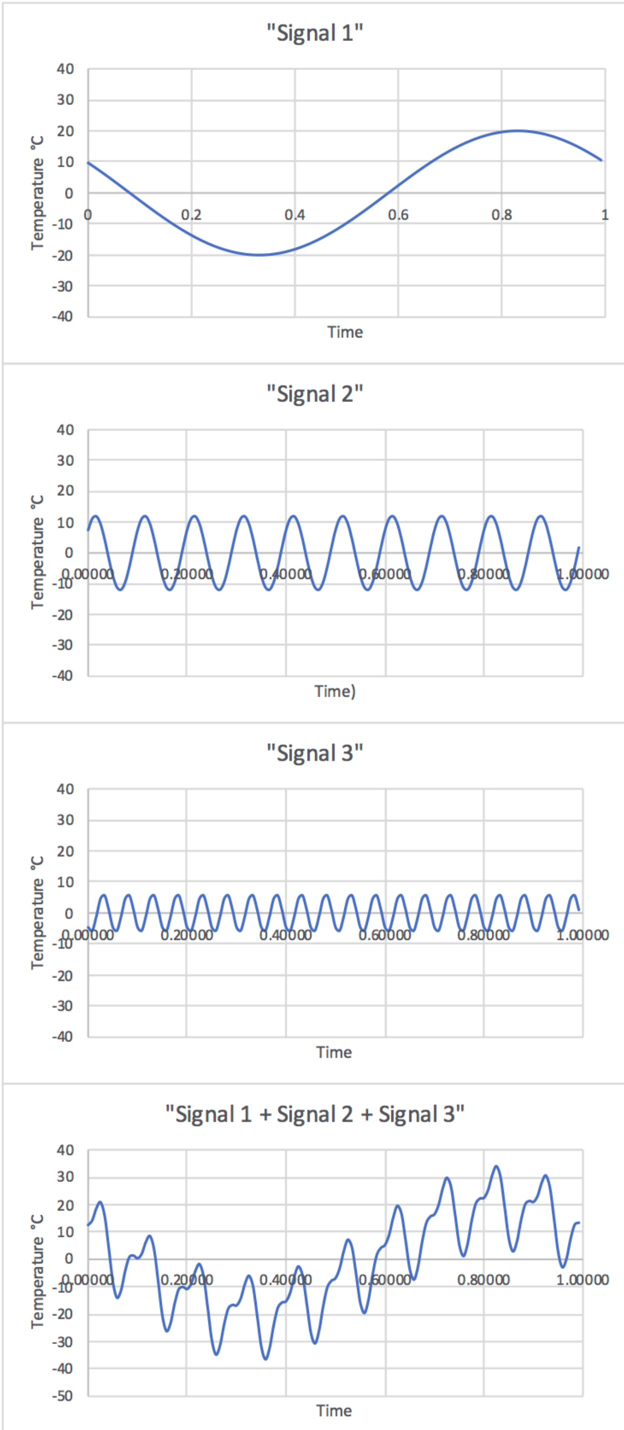

Figure 6A: Visual Aid: Illustrating how the frequency components of the various cycles add up to give the combined air temperature signal we measure.

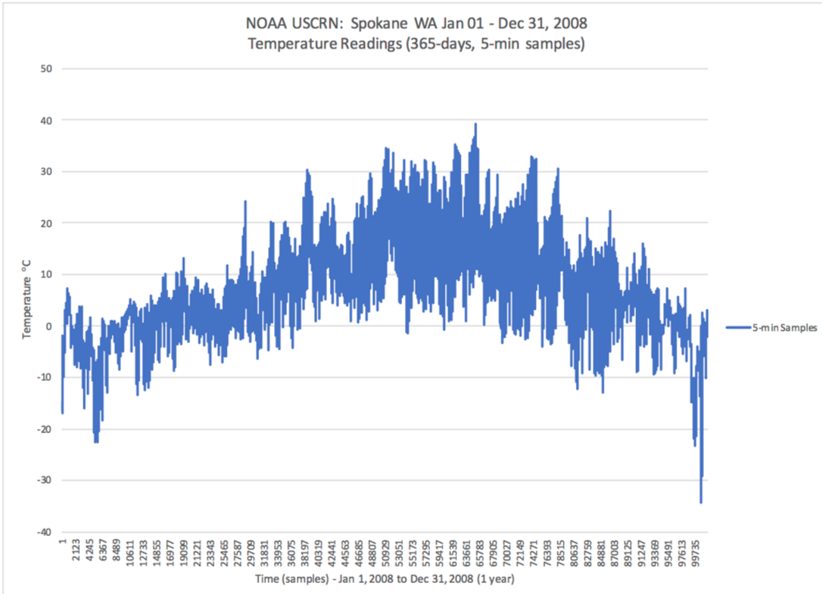

Figure 6B: NOAA USCRN Data for Spokane WA Jan 01 – Dec 31, 2008

Nyquist Compliant Sampling

All of the blue spectral lines in Figure 5A tell us just what frequencies are contained in our example signal. 5-cycles/day is the highest frequency component and therefore according to Nyquist we must sample at a rate which is 2x that or 10-samples/day (or greater), in order to avoid error in our measurement. Figure 7 shows what happens in the frequency domain if we sample at a rate of 12-samples/day. **Through a study of signal analysis, we understand that the process of sampling a signal will create a "spectral image" of the signal, and this spectral image will be centered at the sample rate.** This is an important phenomenon to understand, although it is not intuitive. In Figure 7, the blue lines show our air temperature signal spectrum. This is the same spectrum from Figure 5A. The green and red lines show the **"spectral images"** that our sampling process creates. Note that the red and green images look exactly like the blue one, but these images are shifted up and down by 12-cycles/day (the sample rate). Also note that there is no overlap of either the red or green with the blue lines. This is the critical reason we sample at or above the Nyquist Rate. When we sample below the Nyquist Rate, then the spectrums of the images overlap the spectrum of the original signal. This overlapping is called **"aliasing"**. Aliasing is what causes the error in our measurement and it is undesirable. **Basically, the energy contained in the overlapping spectrum adds to the signal we are sampling and error gets introduced to our measurement. When we sample above the Nyquist Rate, we have no aliasing and no sampling induced error in our measurement.** Figure 7 is an example of a properly sampled signal without aliasing. The sampling complies with the Nyquist Sampling Theorem.

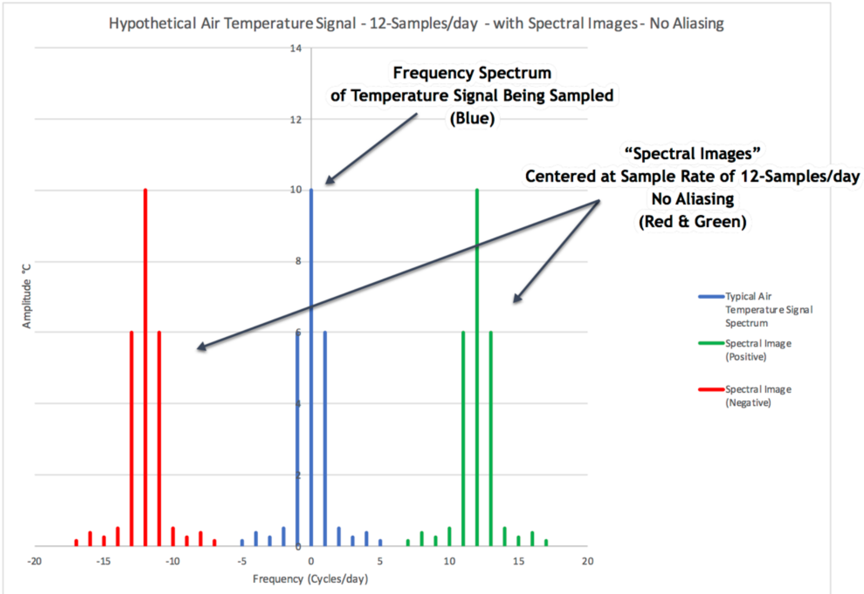

Figure 7: Example Air Temperature Signal – Sampled at 12-samples/day

Violating Nyquist

Next, we will consider an example that violates Nyquist. The historical method of the instrumental temperature record uses 2-samples/day (Tmax and Tmin), so we will examine the spectrum from Figure 5A again, but this time sampled at a rate of 2-cycles/day. Figure 8 shows with blue lines, the same spectrum from Figure 5A. While it is much more difficult to discern visually at first, the red and green spectral images are only shifted up and down by 2-cycles/day. This is because the spectral images are always centered at the sample frequency. In this case, it is clear that there is significant overlap between the spectral images and the original signal spectrum. The red and green lines show up on top of and add to the blue lines. This is the energy from the aliased spectral images corrupting our measurement. Compare the spectrum in Figure 8 to Figure 5A and notice the difference in amplitude at each line. The daily-signal and long-term signal show an alias error of 6.25 °C and 1 °C respectively. **For the case of sampling at a rate of 2-samples/day, the spectral content near 2-cycles/day aliases and corrupts the long-term trends. The spectral content at 1 and 3-cycles/day aliases and corrupts the daily-signal.**

Now, it is important to understand that these frequency charts show the **amplitude** of the frequency components but does not show the signal **"phase"**. The phase relationship between the signal and the aliased image can cause the error to completely add, completely subtract or partially add or subtract. Figure 9 generically illustrates these 3 scenarios. The leftmost column of the image shows how the error completely adds to the signal. The middle column shows how 2 signals completely out of phase can subtract or "cancel". The rightmost column shows

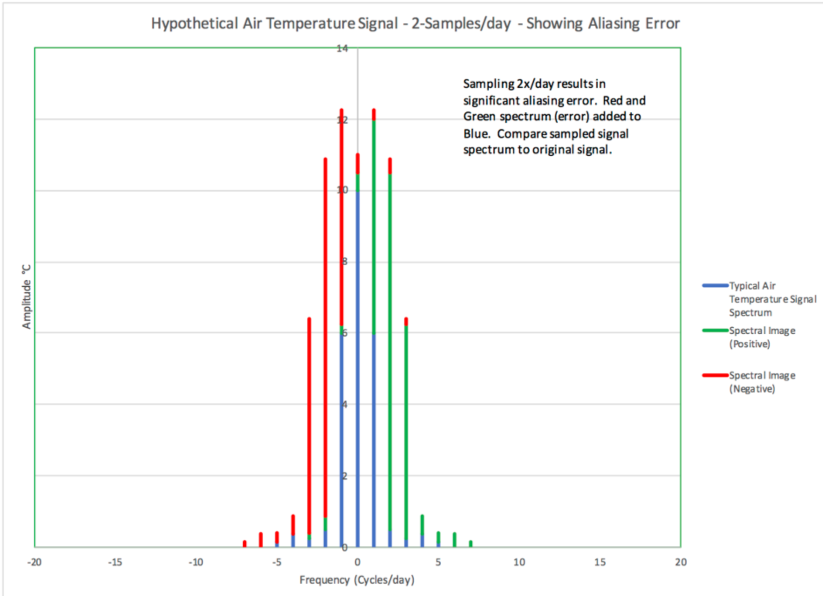

Figure 8: Example Surface Air Temperature Signal – Sampled at 2-samples/day

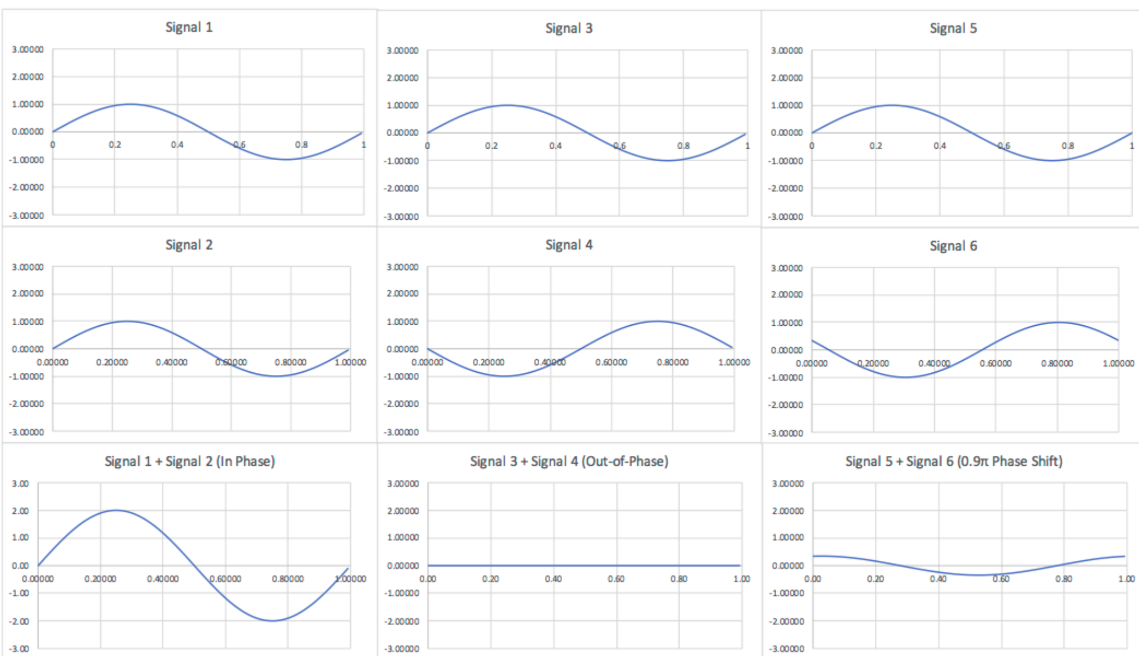

Figure 9: Effect of Phase Relationship to Sampling Error

how signals slightly out of phase can result in an amplitude that is not a simple addition or subtraction of the amplitudes of the signal and image. Figure 8 proves that sampling below Nyquist creates error in the sampled signal but does not tell us exactly how much that error shows up in the time domain.

Some final thoughts on Nyquist and the frequency content of real-world air temperature signals must be communicated. Real-world signals are not limited in frequency. Their frequency content can go on to infinity. This presents a challenge to proper sampling, but one that can be addressed with good system engineering. When air temperature is measured electronically, electrical filters are used to reduce the frequency components that are beyond the specified bandwidth B, thus reducing potential aliasing. Another method of dealing with real-world signals is to sample at a much faster rate. The faster we sample the farther in frequency we space the spectral images, significantly reducing aliasing from undesired frequencies above bandwidth B. This is how Nyquist is applied practically. In the real-world, a small amount of aliasing always exists when sampling, but careful engineering of the system will allow sampling to yield near perfect results toward our goals. When we measure a thermometer by eye, there is no real way to filter out higher frequencies. Also, it is not practically possible for people to read thermometers at a rate that satisfies Nyquist. Sampling must be automated.

Next, we will examine the more intuitive time domain representation of real world air temperature signals obtained from NOAA's USCRN (US Climate REFERENCE Network).

NOAA USCRN and Nyquist

NOAA, in their **USCRN** (US Climate Reference Network) has determined that it is necessary to sample at **4,320-samples/day** to practically implement Nyquist. 4,320-samples/day equates to 1-sample every 20 seconds. **This is the practical Nyquist sample rate.** NOAA averages these 20-second samples to 1-sample every 5 minutes or 288-samples/day. NOAA only publishes the 288-sample/day data (not the 4,320-samples/day data), so to align with NOAA the rate will be referred to as "288-samples/day" (or "5-minute samples"). (Unfortunately, NOAA creates naming confusion with their process of averaging down to a slower rate. It should be understood that the actual rate is 4,320-samples/day.) This rate can only be achieved by automated sampling with electronic instruments. Most of the instrumental record is comprised of readings of mercury max/min thermometers, taken long before automation was an option. Today, despite the availability of automation, the instrumental record still uses Tmax and Tmin (effectively 2-samples/day) instead of a Nyquist compliant sampling. The reason for this is to maintain compatibility with the older historical record. However, with only 2-samples/day the instrumental record is highly aliased. It will be shown in this paper that the historical method introduces significant error to mean temperatures and long-term temperature trends.

NOAA's USCRN is a small network that was completed in 2008 and it contributes very little to the overall instrumental record. However, the USCRN data provides us a special opportunity to compare a high-quality version of the historical method to a Nyquist compliant method. The

Tmax and Tmin values are obtained by finding the highest and lowest values among the 288 samples for the 24-hour period of interest.

NOAA USCRN Examples to Illustrate the Effect of Violating Nyquist on Mean Temperature

The following example will be used to illustrate how the amount of error in the mean temperature increases as the sample rate decreases. Figure 10 shows the temperature as measured at Cordova AK on Nov 11, 2017, using the NOAA USCRN 5-minute samples.

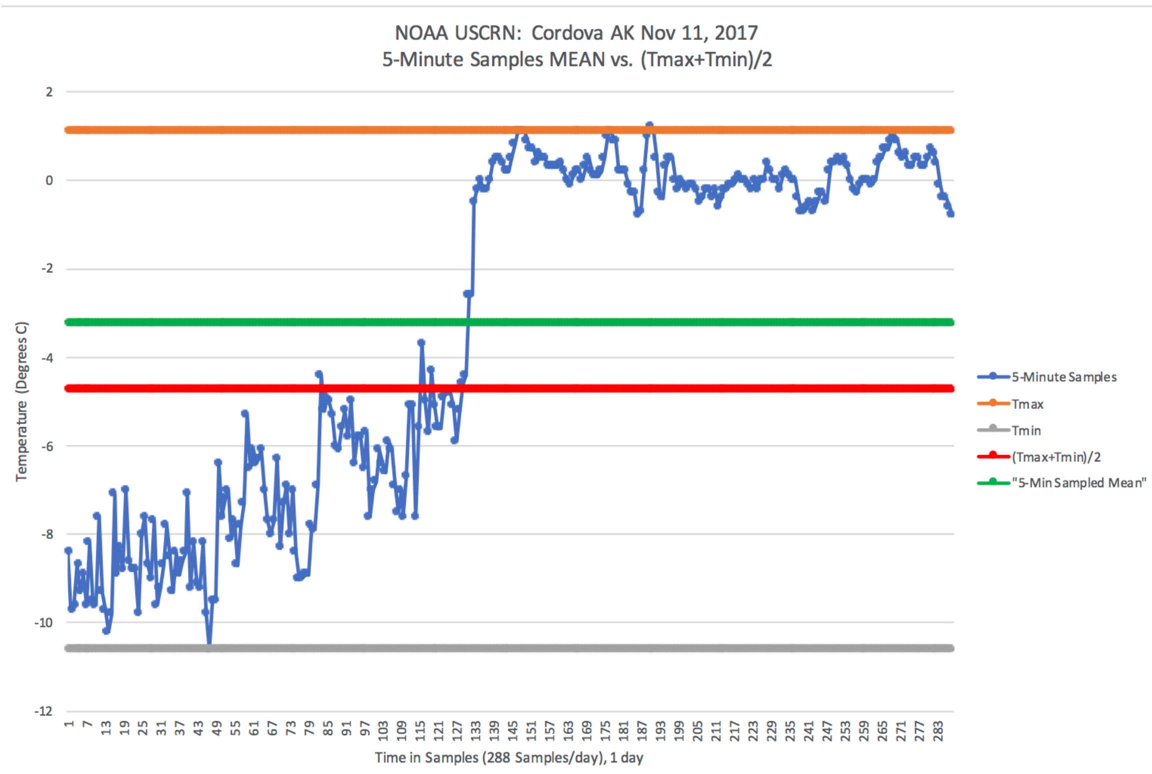

Figure 10: NOAA USCRN Data for Cordova, AK Nov 11, 2017

The blue line shows the 288 samples of temperature taken that day. It shows 24-hours of temperature data. The green line shows the correct and accurate daily mean temperature that is calculated by summing the value of each sample and then dividing the sum by the total number of samples. Temperature is not heat energy, but it is used as an approximation of heat energy. To that extent, the mean (green line) and the daily-signal (blue line) deliver the exact same amount of heat energy over the 24-hour period of the day. The correct mean is -3.3 °C. Tmax is represented by the orange line and Tmin by the grey line. These are obtained by finding the highest and lowest values among the 288 samples for the 24-hour period. The mean calculated from (Tmax+Tmin)/2 is shown by the red line. **(Tmax+Tmin)/2 yields a mean of -4.7 °C, which is a 1.4 °C error compared to the correct mean.**

Using the same signal and data from Figure 10, Figure 11 shows the calculated temperature means obtained from progressively decreased sample rates. These decreased sample rates can be obtained by dividing down the 288-sample/day sample rate by a factor of 4, 8, 12, 24, 48, 72 and 144. Therefore, the sample rates will correspond to: 72, 36, 24, 12, 6, 4 and 2-samples/day respectively. By properly discarding the samples using this method of dividing down, the net effect is the same as having sampled at the reduced rate originally. The corresponding aliasing that results from the lower sample rates, reveals itself as shown in the table in Figure 11.

NOAA USCRN: Cordova AK, Nov 11, 2017				
<--- Decreasing Sample Rate	Samples/day	Tmean C	Error C	<--- Increasing Mean Error
	288	-3.3	0	
	72	-3.2	-0.1	
	36	-3.4	0.1	
	24	-3.4	0.1	
	12	-3.8	0.5	
	6	-4.1	0.8	
	4	-4.0	0.7	
	2	-4.0	0.7	
	(Tmax+Tmin)/2	-4.7	1.4	

Figure 11: Table Showing Increasing Mean Error with Decreasing Sample Rate

It is clear from the data in Figure 11, that as the sample rate decreases below Nyquist, the corresponding error introduced from aliasing increases. It is also clear that 2, 4, 6 or 12-samples/day produces a very inaccurate result. 24-samples/day (1-sample/hr) up to 72-samples/day (3-samples/hr) may or may not yield accurate results. It depends upon the spectral content of the signal being sampled. NOAA has decided upon 288-samples/day (4,320-samples/day before averaging) so that will be considered the current benchmark standard. Sampling below a rate of 288-samples/day will be (and should be) considered a violation of Nyquist.

It is interesting to point out that what is listed in the table as 2-samples/day yields 0.7°C error. But (Tmax+Tmin)/2 is also technically 2-samples/day with an error of 1.4°C as shown in the table. How can this be possible? It is possible because (Tmax+Tmin)/2 is a special case of 2-samples per day because these samples are not spaced evenly in time. The maximum and minimum temperatures happen whenever they happen. When we sample properly, we sample according to a "clock" – where the samples happen regularly at exactly the same time of day. The fact that Tmax and Tmin happen at irregular times during the day causes its own kind of sampling error. It is beyond the scope of this paper to fully explain, but this error is related to what is called "clock jitter". It is a known problem in the field of signal analysis and data acquisition. 2-samples/day, regularly timed, would likely produce better results than finding the maximum and minimum temperatures from any given day. The instrumental temperature record uses the absolute worst method of sampling possible – resulting in maximum error.

Figure 12 shows the same daily temperature signal as in Figure 10, represented by 288-samples/day (blue line). Also shown is the same daily temperature signal sampled with 12-samples/day (red line) and 4-samples/day (yellow line). From this figure, it is visually obvious that a lot of information from the original signal is lost by using only 12-samples/day, and even more is lost by going to 4-samples/day. This lost information is what causes the resulting mean to be incorrect. This figure graphically illustrates what we see in the corresponding table of Figure 11. Figure 12 explains the sampling error in the time-domain. It corresponds to the aliasing we saw previously in the frequency domain, in Figure 8.

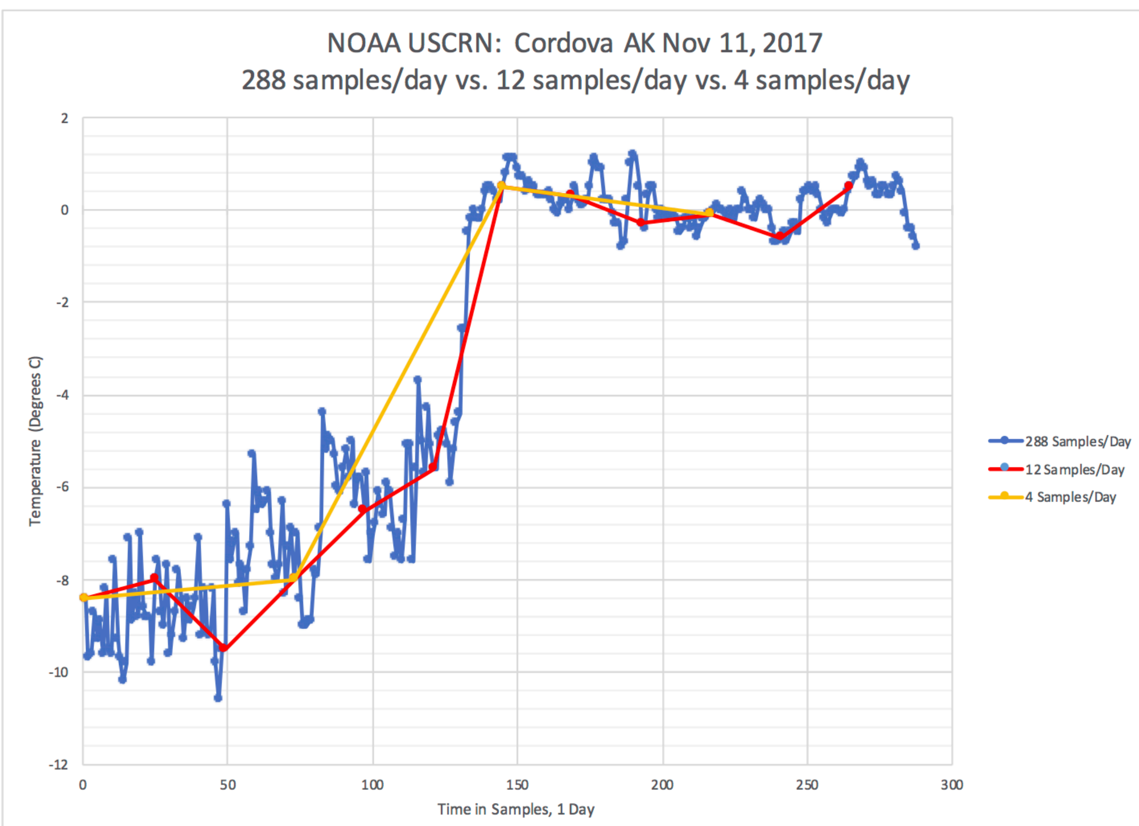

Figure 12: NOAA USCRN Data for Cordova, AK Nov 11, 2017: Decreased Detail from 12 and 4-Samples/Day Sample Rate – Time-Domain

Figure 13 shows the daily mean error between the USCRN 288-samples/day method and the historical method, as measured over 365 days at the Boulder CO station in 2017. Each data point is the error for that particular day in the record. **We can see from Figure 13 that (Tmax+Tmin)/2 yields daily errors of up to ± 4 °C. Calculating mean temperature with 2-samples/day rarely yields the correct mean.**

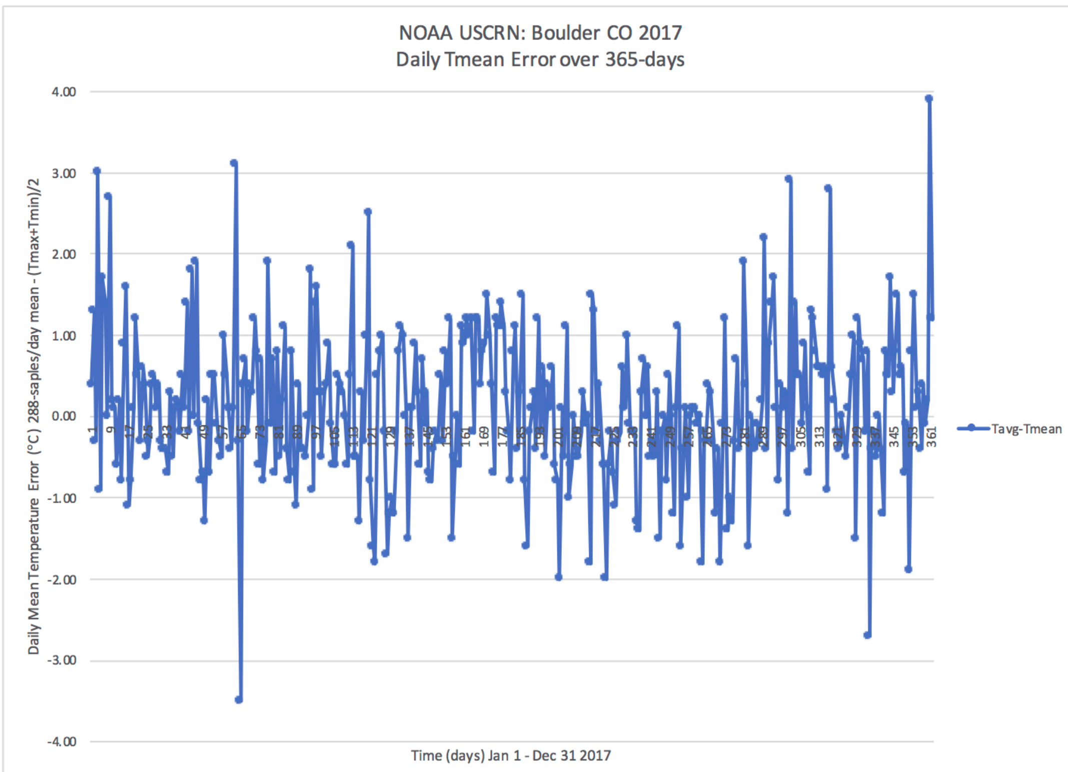

Figure 13: NOAA USCRN Data for Boulder CO – Daily Mean Error Over 365 Days

Let's look at another example, similar to the one presented in Figure 10, but over a longer period of time. Figure 14 shows (in blue) the 288-samples/day signal from Spokane WA, from Jan 13 – Jan 22, 2008. Tmax (avg) and Tmin (avg) are shown in orange and grey respectively. The (Tmax+Tmin)/2 mean is shown in red (-6.9 °C) and the correct mean calculated from the 5-minute sampled data is shown in green (-6.2 °C). The (Tmax+Tmin)/2 mean has an error of 0.7 °C over the 10-day period.

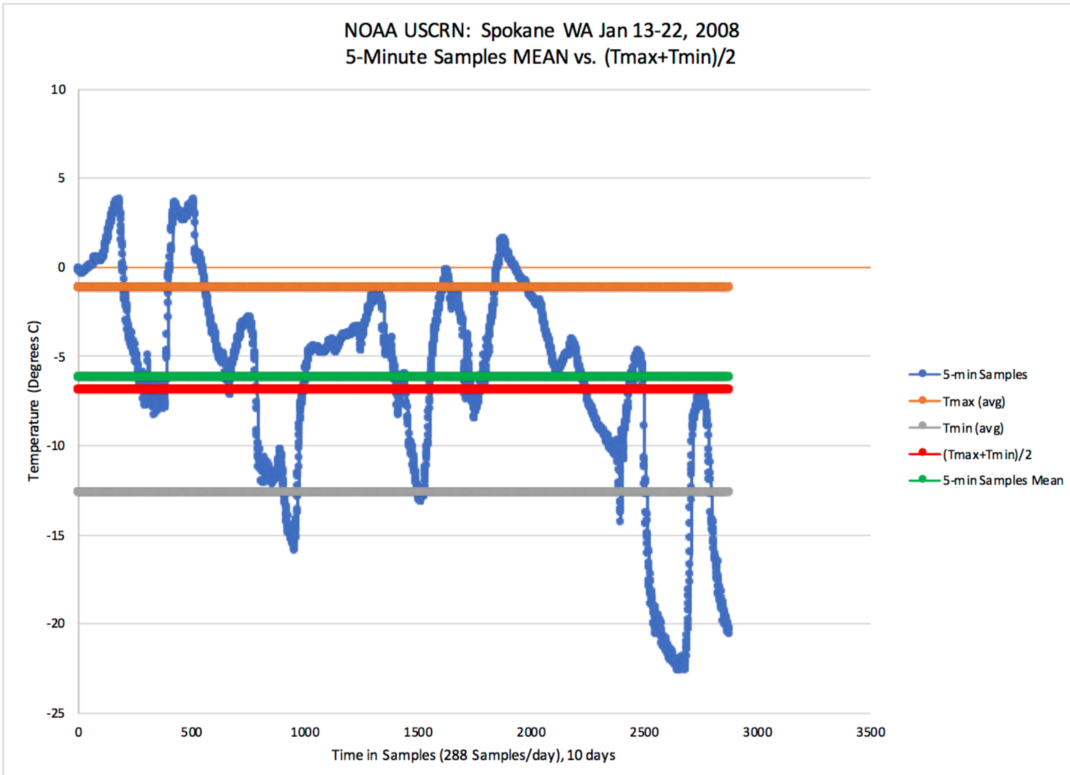

Figure 14: NOAA USCRN Data for Spokane, WA – Jan13-22, 2008

The Effect of Violating Nyquist on Temperature Trends

Finally, we need to look at the impact of violating Nyquist on temperature trends. In Figure 15, a comparison is made between the linear temperature trends obtained from the historical and Nyquist compliant methods using NOAA USCRN data for Blackville SC, from Jan 2006 – Dec 2017. We see the trend derived from the historical method (orange line) starts approximately 0.2 °C warmer and has a 0.24 °C/decade warming bias compared to the Nyquist compliant method (blue line). Figure 16 shows the trend bias or error (°C/Decade) for 26 stations in the USCRN over a 7-12 year period. The 5-minute samples data gives us our reference trend. The trend bias is calculated by subtracting the reference from the (Tmaxavg+Tminavg)/2 derived trend. Almost every station exhibits a warming bias, with a few exhibiting a cooling bias. The largest warming bias is 0.24 °C/decade and the largest cooling bias is -0.17 °C/decade, with an average warming bias across all 26 stations of 0.06 °C/decade. As was stated earlier, the calculated global average warming trend for the period 1880-2012 is 0.064 ± 0.015 °C per decade. If we look at the more recent period that contains the controversial "Global Warming Pause", then using data from

Wikipedia, we get the following warming trends depending upon which year is selected for the starting point of the "pause":

1996: 0.14°C/decade
1997: 0.07°C/decade
1998: 0.05°C/decade

While no conclusions can be made by comparing the trends over 7-12 years from 26 stations in the USCRN to the currently accepted long-term or short term global average trends, it can be instructive. It is clear that using the historical method to calculate trends yields a trend error and this error can be of a similar magnitude to the claimed trends. Therefore, it is reasonable to call into question the validity of the trends. There is no way to know for certain, as the bulk of the instrumental record does not have a properly sampled alternate record to compare it to. But it is a mathematical certainty that every mean temperature and derived trend in the record contains significant error if it was calculated with 2-samples/day.

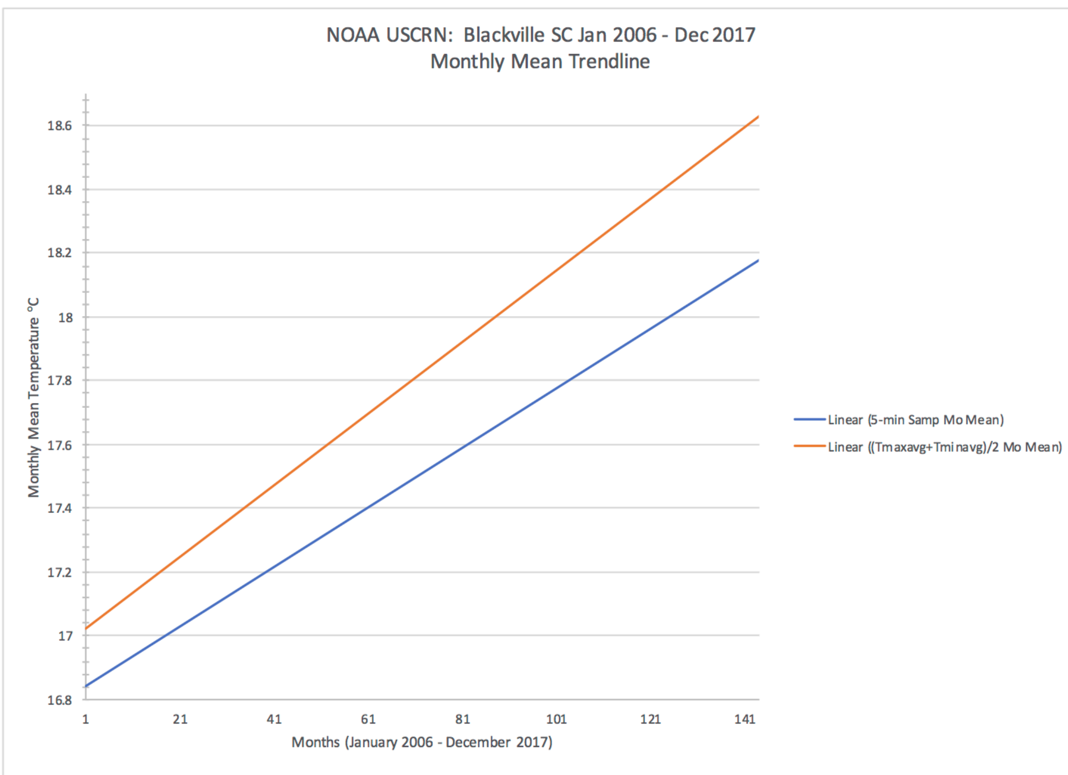

Figure 15: NOAA USCRN Data for Blackville, SC – Jan 2004-Dec 2017 – Monthly Mean Trendlines

NOAA USCRN Station	Years (inclusive)	Trend Bias (°C/Decade)
Boulder (CO)	2006-2017	0.15
Newton SW (GA)	2007-2017	0.07
Monahans (TX)	2010-2017	0.19
Montrose (CO)	2007-2017	0.11
Spokane (WA)	2008-2017	-0.17
Fallbrook (CA)	2009-2017	0.01
Darrington (WA)	2007-2017	0.04
Limestone (ME)	2007-2017	0.09
Baker (NV)	2007-2017	0.18
Port Alsworth (AL)	2011-2017	-0.05
Bedford (IN)	2008-2017	0.04
LaFayette (LA)	2007-2017	0.09
Blackville (SC)	2006-2017	0.24
Crossville (TN)	2006-2017	0.10
Brigham City (UT)	2008-2017	0.07
Elkins (WV)	2006-2017	0.07
Sioux Falls (SD)	2007-2017	0.06
Tucson (AZ)	2007-2017	0.07
Durham N (NH)	2006-2017	-0.06
Asheville SSW (NC)	2008-2017	0.12
Cape Charles (VA)	2006-2017	0.02
Lincoln (NE)	2007-2017	0.06
Stovepipe (CA)	2006-2017	0.07
Moose (WY)	2007-2017	0.14
Wolf Point ENE (MT)	2007-2017	-0.02
Palestine (TX)	2007-2017	0.03

Figure 16: Trend Bias (°C/Decade) for 26 Stations in USCRN

Conclusions

1. Air temperature is a signal and therefore, it must be measured by sampling according to the mathematical laws governing signal processing. Sampling must be performed according to The Nyquist-Shannon Sampling Theorem.
2. The Nyquist-Shannon Sampling Theorem has been known for over 80 years and is essential science to every field of technology that involves signal processing. Violating Nyquist guarantees samples will be corrupted with aliasing error and the samples will not represent the signal being sampled. Aliasing cannot be corrected post-sampling.
3. The Nyquist-Shannon Sampling Theorem requires the sample rate to be greater than 2x the highest frequency component of the signal. Using automated electronic equipment and computers, NOAA USCRN samples at a rate of 4,320-samples/day (averaged to 288-samples/day) to practically apply Nyquist and avoid aliasing error.
4. The instrumental temperature record relies on the historical method of obtaining daily Tmax and Tmin values, essentially 2-samples/day. Therefore, the instrumental record violates the Nyquist-Shannon Sampling Theorem. The spectral content at 1 and 3-cycles/day aliases and corrupts the daily-signal and spectral content near 2-cycles/day aliases and corrupts the long-term signal.
5. NOAA's USCRN is a high-quality data acquisition network, capable of properly sampling a temperature signal. The USCRN is a small network that was completed in 2008 and it contributes very little to the overall instrumental record, however, the USCRN data provides us a special opportunity to compare analysis methods. A comparison can be made between temperature means and trends generated with Tmax and Tmin versus a properly sampled signal compliant with Nyquist.
6. Using a limited number of examples from the USCRN, it has been shown that using Tmax and Tmin as the source of data can yield the following error compared to a signal sampled according to Nyquist:
    a. Mean error that varies station-to-station and day-to-day within a station.
    b. Mean error that varies over time with a mathematical sign that may change (positive/negative).
    c. Daily mean error that varies up to +/-4°C.
    d. Long term trend error with a warming bias up to 0.24°C/decade and a cooling bias of up to 0.17°C/decade.
7. The full instrumental record does not have a properly sampled alternate record to use for comparison. More work is needed to determine if a theoretical upper limit can be calculated for mean and trend error resulting from use of the historical method.
8. The extent of the error observed with its associated uncertain magnitude and sign, call into question the scientific value of the instrumental record and the practice of using Tmax and Tmin to calculate mean values and long-term trends.

Reference section:

This USCRN data can be found at the following site:
https://www.ncdc.noaa.gov/crn/qcdatasets.html

NOAA USCRN data for Figure 10 is obtained here:
https://www1.ncdc.noaa.gov/pub/data/uscrn/products/subhourly01/2017/CRNS0101-05-2017-AK_Cordova_14_ESE.txt

NOAA USCRN data for Figure 13 is obtained here:
https://www1.ncdc.noaa.gov/pub/data/uscrn/products/daily01/2017/CRND0103-2017-AK_Cordova_14_ESE.txt

NOAA USCRN data for Figure 14 is obtained here:
https://www1.ncdc.noaa.gov/pub/data/uscrn/products/subhourly01/2008/CRNS0101-05-2008-WA_Spokane_17_SSW.txt

NOAA USCRN data for Figure 15 is obtained here:
https://www1.ncdc.noaa.gov/pub/data/uscrn/products/monthly01/CRNM0102-SC_Blackville_3_W.txt